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Mixture design of experiments on portfolio optimisation of power generation

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Abstract: A methodology for obtaining an optimal portfolio for the generation of electricity at the lowest cost and risk is proposed. This methodology uses a mixture design of experiments (MDEs) as a strategy for building nonlinear models of risk and cost in portfolio optimisation for the generation of electricity. The result is compared with the traditional theory of Markowitz mean–variance (MVP). The following characteristics are also presented in this study: the seasonality and volatility of the time series were manipulated using moving windows and computational replicas in MDE; desirability functions were used to optimise multiple variables, leading to lower cost and risk; Shannon entropy index was used to handle better portfolio diversification. A case study based on the energy market of the state of California was used to illustrate the proposal. The results show that this methodology facilitates decision making.

1 Introduction

Financial investors find portfolios that produce efficient results under various economic conditions by using what is known as portfolio theory. An efficient portfolio is a combination of investments that maximises expected return while minimising risk. Such a combination is known as an optimal portfolio [1].

There are several studies of portfolio theory in applications for optimising the electricity generation mix of a particular region. Among these studies is the Markowitz mean-variance model, establishing the optimal strategy for minimising the risk and maximising the return [2]. Two others include the variance–Skewness–Kurtosis-based portfolio optimisation [3] and the use of genetic algorithms and multi-objective optimisation [4].

How to allocate different assets in a profitable portfolio is one of the major interesting issues in many areas including the electricity market ([5–10]).

Portfolio theory can also be used to diversify power generation, minimising cost and risk, while taking into consideration each technology used in power generation. In [5], researchers created a model of the cost and risk for electric power generation and an efficient frontier was established through Markowitz mean-variance (MVP) theory. Other studies that used MVP portfolio optimisation in the energy market, such as [11], can be highlighted. Such a work uses MVP to determine strategic positions in balancing the energy market as well as in identifying corresponding economic incentives in an analysis of the German balancing energy demand. In [12], the authors combined the MVP along with Monte Carlo simulations to identify, using their investment returns, optimal-based load generation portfolios for large electricity producers in liberalised electricity markets.

Designing an optimal portfolio has been the focus of many papers. Among them can be cited the Markowitz mean-variance model, establishing the optimal strategy for minimising risk and maximising return [2]. Two others include minimising the variance, by estimating the covariance matrix [13] and using an estimator for the covariance matrix [14]. In [15], DeMiguel introduces the restriction of the weight vector of the portfolio standard. One can also find an optimal portfolio using design of experiments [16]. Thus, the objective of this study is to use the cost modelling and risk defined in [5] to find, through a mixture design of experiments (MDEs), with moving windows and computational replicas in MDE, an optimal portfolio for generating electricity.

The value at risk (VaR) can be used to calculate the risk. VaR into account not only the individual risk of each asset, expressed statistically by variance of returns, but also the relationships between the various assets given by correlations [16]. According to [4], this metric describes the loss that can occur over a given period with a certain level of confidence, due to exposure to market risk. The risk can also be calculated using the metric value in conditional risk, which identifies the average loss considering all instances where the return is lower than the VaR [17]. In this paper, standard deviation will be used, which is also considered a good metric for defining risk by many researchers.

The advantage of using the proposed method is that there is no need to model the time series or make predictions to capture its behaviour. Then, seasonality and volatility data are integrated into the portfolio optimisation. This makes the process easier, enabling one to avoid concepts such as nonlinearity, volatility etc. In addition, a careful review of the literature on electricity generation portfolios using MDE produced no specific methodology available, which emphasises the value of the methodology developed in this article.

The behaviour of the times series is not a concern, since this is handled by the moving windows and replica. The use of MDE is well established in many fields. In Chemistry, Engineering, Physics, etc., MDE has several applications. In Finance, Economics, Computer Science and Mathematics, MDE was rarely observed [18–20]. This new methodology for obtaining an optimal portfolio combines some concepts (MDE, time series, entropy etc) in a unique way, accounting for a unique decision-making application in the electricity market.

This paper is organised as follows: Section 2 presents fundamental concepts of portfolio optimisation from the theory of Markowitz mean-variance (MVP). Concepts of optimisation based on MDE are also covered, in addition to the Shannon entropy index. This section also presents the concepts of moving windows and replicas. The finishing section will present the desirability function. A case study is used as an application of the proposed methodology and the results are presented in Section 3. Finally, the last section presents some concluding remarks.

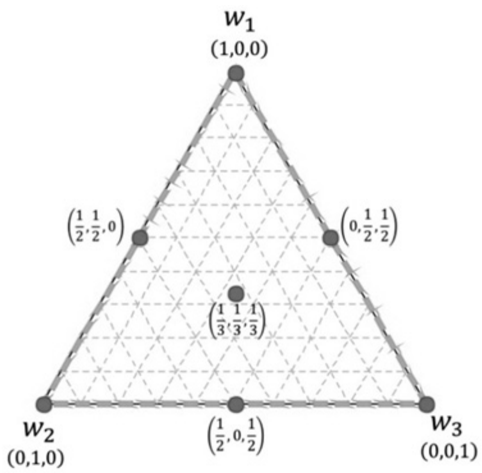

Fig. 1 *Simplex coordinate system with three components*

2 Portfolio optimisation based on mixture design of experiments

2.1 Portfolio optimisation

Portfolio theory provides information, based on a risk-return analysis, to aid decision-making related to selecting investments. To help make this process more efficiently, researchers have developed some models of portfolio optimisation. The investor's interest is, of course, to maximise the return and minimise the risk.

In portfolio problems, one should consider three dimensions: the expected return of each asset making up the portfolio, the risk each asset brings to the portfolio and the amount invested in each asset [16].

Harry Markowitz presented a theory in 1952 (which can be found in [21]), called mean-variance (MVP). MVP allows for the creation of portfolios with minimum variance and the maximum expected return. The expected return (denoted by μ_c) is the sum of the weighted average of each asset in the proportions of investment as weight (denoted by w_i), thus $\mu_c = \sum_i^n w_i\mu_i$. The variance of the distribution indicates the amplitude of the possible outcomes around the mean and can be written as the expected value of the linear combination involving a covariance term σ_{ij}, which measures how the two active returns are correlated. The variance is given by $\sigma^2 = \sum_{i=1}^n w_i^2\sigma_i^2 + \sum_{j\neq i} w_i w_j \sigma_{ij}$.

Considering the weights and values of the MVP model as proportions of a mixture whose sum is a unity, or restricted to a certain limit, the MVP can also be characterised as an MDE.

MDE is a special class of response surface experiments in which the investigational product consists of multiple components or ingredients. The response is a function of the proportions of the different ingredients of the mixture. These proportions are non-negative and are expressed as fractions of the total mixture. The sum must be equal to one [22].

The space formed by mixture experiment components is described as a system of simplex coordinates. A uniform distribution of these coordinates on the simplex is known as a lattice [22].

As shown in [22], the lattice can correspond to a specific polynomial equation. For example, a polynomial model of degree m to a mixture of q components, called $\{q, m\}$ simplex-lattice, consists of a coordinated set of points that define the proportions of each ingredient, distributed as as follows.

$$w_i = 0,\ \frac{1}{m},\ \frac{2}{m},\ \ldots,\ 1 \tag{1}$$

Table 1 Common MDE models

Terms	Components in the model
linear	β_1A + β_2B + β_3C
quadratic	linear + β_4AB + β_5AC + β_6BC
special cubic	quadratic + β_7ABC
full cubic	special cubic + β_8AB(A – B) + β_9AC(A – C) + β_{10}BC(B – C)
special quartic	quadratic + β_7AABC + β_8ABBC + β_9ABCC
full quartic	special quartic + β_{10}AB(A – B) + β_{11}AC(A – C) + β_{12}BC(B – C) + β_{13}AB(A – B)^2 + β_{14}AC(A – C)^2 + β_{15}BC(B – C)^2
inverse	$\beta_1$1/A + $\beta_2$1/B + $\beta_3$1/C

To illustrate, consider a mixture of three components, $q = 3$; the degree of the polynomial is two, $m = 2$. Therefore, the proportion of each component takes values $w_i = 0, (1/2)$ and 1 for all $i = 1, 2$ and 3 and $\{3, 2\}$ simplex-lattice consists of six points, presented in (2), on the border of the triangle shown in Fig. 1. (see (2))

$$(w_1, w_2, w_3) = (1, 0, 0), (0, 1, 0),\ (0, 0, 1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right), \left(0, \frac{1}{2}, \frac{1}{2}\right). \tag{2}$$

The first three components are the nodes and represent the pure mixture; the last three terms represent mixtures of two components [16]. The central point, called the centroid, is the point where the mixture has equal proportions of each component [16].

The relationship between response variables and the relative proportion of q components can be defined by a polynomial of degree m, which is usually linear, quadratic or cubic, depending on the objectives of response. Mixture designs include many types of model terms. Table 1 exemplifies these terms considering a mixture design with three components A, B and C. The β coefficient in the regression equation uses least squares estimation [22].

One problem found with MDE is a tendency to generate highly concentrated portfolios in a few assets with poor performance outside the sample. This condition contradicts the concept of diversification. Many authors, trying to improve portfolio diversification, have used a maximisation of the Shannon entropy index, shown next.

2.2 Shannon entropy

Diversify the portfolio is not to attribute most of the weight to just a few assets, because that would increase their risk. Adding an upper limit restricting the weights of assets, according to [14], is the most common and simplest way of controlling the diversity of a portfolio. The portfolio can also be diversified by imposing a lower limit restriction on the weights. In this section, the constraints that are found in the literature [23] were defined.

Consider the expected return μ_c of n active, the variance σ^2 and the covariance σ_{ij}. To optimise a portfolio is to maximise the following objective function $f(w_i)$

$$0 \le w_i \le 1, \quad \text{for } i = 1 \text{ to } n \tag{3}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{4}$$

Constraint (4) ensures non-negative weights, which is necessary to calculate the entropy as discussed below, and constraint (5) requires the investment of all capital.

The entropy constraint adds a lower bound on the entropy of a portfolio. According to [24], the author defines entropy as a discrete set of probabilities $p_1, p_2, \ldots, p_n$ as

$$\text{Ent} = -\sum_{i=1}^{n} p_i(x)\log p_i(x) \tag{5}$$

In a similar manner, the entropy of a continuous distribution with a density distribution $p(x)$ is defined by

$$\text{Ent} = -\int_{-\infty}^{\infty} p(x)\log p(x)\mathrm{d}x \tag{6}$$

As in a portfolio, the weights for each asset i and w_i are proportions; then $p(w_i)$ is a discrete probability distribution. Thus, from (6), it can be concluded that the entropy constraint adds a lower bound L_E on the entropy Ent of a portfolio w_i, as defined below [25]

$$\text{Ent} = -\sum_{i=1}^{n} p_i(x)\log p_i(x) = -\sum_{i=1}^{n} w_i \log w_i \ge L_\text{E} \tag{7}$$

In the least diverse scenario where only one component of w is 1 and the rest of the components of w are 0, Ent reaches its minimum $-1 \times \log\ 1 = 0$. In the most diverse scenario that $w_i = (1/n)$ for all i, Ent reaches its maximum $-n\big((1/n)\log(1/n)\big) = \log\ n$. Therefore, L_E is between the interval $\big[0,\ \log\ n\big]$. Since a larger Ent indicates better diversity, the entropy constraint uses a lower bound L_E within the same interval to control the diversity of w_i from being too low [14].

2.3 Moving windows and replicas

In this section, the basics of moving windows and replicas are presented. The concept of autocorrelation function (ACF) is firstly explored. According to [26], to determine a proper model for a given time series data, it is necessary to carry out the ACF analysis. These statistical measures reflect how the observations in a time series relate to each other. For modelling purposes, it is often useful to plot the ACF against consecutive time lags.

Consider a time series y_t. The covariance between y_t and its value at another time period, y_{t+k} is called the auto covariance at lag k, can be defined by

$$\gamma_k = \text{Cov}(y_t, y_{t+k}) = E\big[(y_t - \mu)(y_{t+k} - \mu)\big] \tag{8}$$

The collection of values $\gamma_k,\ k = 0,\ 1,\ 2,\ \ldots$ is defined as the auto covariance function. γ_0 is the variance of the time series, that is $\gamma_0 = \sigma_y^2$. The autocorrelation coefficient at lag k is

$$\rho_k = \frac{E[(y_t - \mu)(y_{t+k} - \mu)]}{\sqrt{E\Big[(y_t - \mu)^2\Big]E[(y_{t+k} - \mu)^2]}} = \frac{\text{Cov}(y_t, y_{t+k})}{\text{Var}(y_t)} = \frac{\gamma_k}{\gamma_0} \tag{9}$$

The collection of the values of $\rho_k,\ k = 0,\ 1,\ 2,\ \ldots$ is called the ACF. Note that by definition $\rho_0 = 1$. As explained by Box and Jenkins [27], the sample ACF plot is useful in modelling a time series of length N. Since ACF is symmetrical regarding lag zero, it is only required to plot the ACF sample for positive lags, from lag one onwards to a maximum lag of about, according to [26, 27], about

$$L = \frac{N}{4} \tag{10}$$

Given a time series, a moving window consists of all possible subsets of data size L. The size L of the window is given by the lag of the ACF, according to (11), where N is the total number of data points, and it is called the replica number of moving windows that scroll the time series, so that all observations are analysed. The step size of the movement of the window is given by the lag of the ACF with 5% significance. In this paper, replicas and moving windows are used to capture the behaviour, such as seasonality and volatility, of the series over time. The moving windows procedure simplifies the whole process because it is not necessary to model the time series.

In design of experiments, the replicas are taken from identical experimental runs but with different characteristics. When the whole design is replicated, the complete set of design points is duplicated. Replicates are used to estimate the variance (experimental error) caused by slightly different experimental conditions. The experimental error serves as a benchmark to determine whether observed differences in the data are statistically different. The design points that would be added to a second-degree three-component simplex-lattice design, as shown in Fig. 1, are exemplified in Table 2.

In prediction models, using replicas increase the accuracy of the model, allowing for the detection of smaller effects, or to enough power to detect a fixed size effect. True replication provides an estimate of the error or noise in its process and may allow for more precision in these estimates.

In summary, the advantage of using moving windows and replicas are

- detect trends faster in series;
- explore best seasonality;
- increase in the degree of freedom for parameter estimation;
- improve the accuracy of the model and enable detection of small effect in the series;

Table 2 Three replicas in MDE

Initial design			One replicate added (total of two replicates)			Two replicates added (total of three replicates)		
A	B	C	A	B	C	A	B	C
1	0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	1	0
0	0	1	0	0	1	0	0	1
0.5	0.5	0	0.5	0.5	0	0.5	0.5	0
0.5	0	0.5	0.5	0	0.5	0.5	0	0.5
0	0.5	0.5	0	0.5	0.5	0	0.5	0.5
0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33
			1	0	0	1	0	0
			⋮	⋮	⋮	⋮	⋮	⋮
			0.33	0.33	0.33	0.33	0.33	0.33
						1	0	0
						⋮	⋮	⋮
						0.33	0.33	0.33

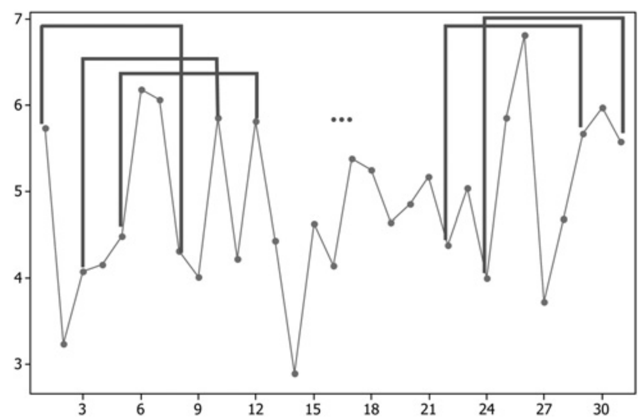

Fig. 2 *Example for moving windows and replicas*

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• provide an estimate of the error allowing for estimates that are more accurate.

Fig. 2 shows an example of a temporal series with 13 replicas and moving window size of 8.

To obtain an optimal portfolio, moving windows and replica are used to capture the behaviour of the time series, while the entropy index is used to diversify it. The desirability function is employed as an optimisation of multiple responses.

2.4 Desirability function

The desirability function involves transforming each estimated response variable $\hat{y}_i$ into a desirable individual value d_i, where $0 \le d_i \le 1$. The individual desirabilities are combined through a simple geometric mean as in (12), or by the geometric mean weight (z_i), given by (13). These weights indicate the importance of each property in relation to others in the multi-objective optimisation process.

$$D = \left(d_1 \times d_2 \times \cdots \times d_k\right)^{(1/k)} \tag{11}$$

$$D = \left(d_1^{z_1} \times d_2^{z_2} \times \cdots \times d_k^{z_k}\right)^{\left(1/\Sigma_i^k z_i\right)} \tag{12}$$

where k is the number of variables and the response value D measures total desirability. The combination of the individual desirability for each level of response and its value lie between the interval [0, 1]

To minimise a response variable (y) through the desirability function given by [28], a transformation of variables was used according to (14). Equation (15) was used to maximise the response variable (y)

$$d[y] = \begin{cases} 0 & \text{se} \quad \hat{y}_i > H_i \\ \left[\dfrac{(H_i - \hat{y}_i)}{(H_i - T_i)}\right]^{\lambda} & \text{se} \quad T_i \le \hat{y}_i \le H_i \\ 1 & \text{se} \quad \hat{y}_i < T_i \end{cases} \tag{13}$$

$$d[y] = \begin{cases} 0 & \text{se} \quad \hat{y}_i < L_i \\ \left[\dfrac{(\hat{y}_i - L_i)}{(T_i - L_i)}\right]^{\lambda} & \text{se} \quad L_i \le \hat{y}_i \le T_i \\ 1 & \text{se} \quad \hat{y}_i > T_i \end{cases} \tag{14}$$

where L_i is the lower limit of desirability, H_i is the upper limit, T_i is the target of the desirability and λ is the parameter of desirability. When $\lambda \sim 1$, equal emphasis is given to the target and limits; when $\lambda \sim 10$, $\hat{y}_i$ assumes a value closer to the target [16]. Fig. 3 illustrates how the weights affect the result.

An MVP using the average μ_c for the return, the variance σ^2 as a risk measure, the entropy Ent to improve portfolio diversification, and adopting the transformations obtained by the desirability function approach to portfolio optimisation based on MDE can be written, based on [16], as

$$\begin{aligned} \text{Max} \quad & D = \sqrt{d_{\mu_c} \times d_{\sigma^2} \times d_{\text{Ent}}} \\ \text{subject to:} \quad & d^{n+1}(y_i) \ge D, \; i = 1, 2, \ldots, k \\ & D \ge 0 \\ & w \in \Omega \end{aligned} \tag{15}$$

$d^{n+1}(y_i)$ being the desirability function y_i in $(n+1)$th iteration; D is the minimum value of desirability set at the beginning of the iterative minimisation model; $w \in \Omega$ denotes the entire region defined early in the process.

Given the discussion above, the MMSE approach proposed in this section may be summarised as follows:

- Determining moving window and replicas:
  Step 1: Compute moving window size and number for replicas using ACF and (11).
  Step 2: Compute the average μ, the risk σ and correlation ρ, for each replica
- Portfolio optimisation
  Step 3: Create the MDE with replicas for return, risk and entropy.
  Step 4: Compute the return, risk and entropy for each replica.
  Step 5: Perform regression analysis to choose the best polynomial that models return, risk and entropy, calculating the coefficients of the polynomial.
  Step 6: Define the limits and target for function desirability.
  Step 7: Acquire the feasible solutions and the portfolio compound.

The proposed methodology is robust enough to deal with huge systems. For this sake, a real power system is employed, as described next.

3 Portfolio optimisation of power electricity generation in the state of California

This section applies the proposed methodology in portfolio optimisation to California's electricity sector to find the best combination for power generation. This combination has the lowest cost and risk. Five technologies are chosen to compose the portfolio: Combined Cycle Standard (natural gas fuel), Integrated Gasification Combined Cycle—IGCC (coal fuel), solar photovoltaic, hydro and geothermal.

The costs and risks can be divided into a fixed part (US$/MW), determined by installed capacity, and a variable part (US$/MWh), determined by electricity generated by a certain plant. Modelling of cost and risk is based on studies presented in [5].

3.1 Mathematical modelling of cost and risk

This approach does not consider the time dimension. All costs are expressed in (US$/MWh) determined from the number of annual operating hours by technology or by (US$/MW) determined by the capacity of the plant.

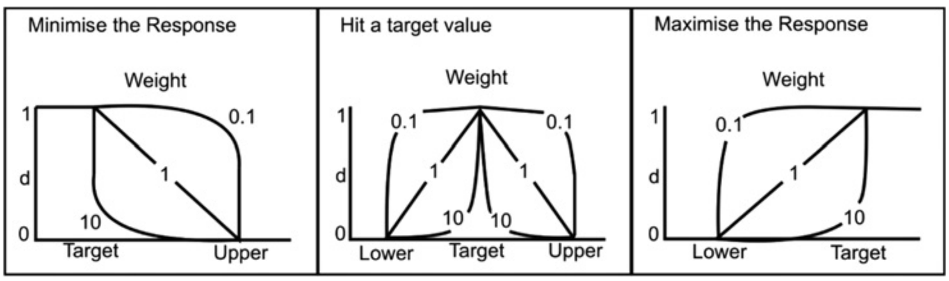

Fig. 3 *Desirability functions for different goals—how Weights affect their shapes*

Table 3 Fixed and variable costs

	Combined cycle standard	IGCC	Solar photovoltaic	Hydro	Geothermal
invest.	28.64	72.98	257.53	93.65	84.76
O&M fix.	3.66	11.98	0.00	4.85	5.94
O&M var.	1.61	9.38	47.03	11.10	11.15

Invest., investment cost (US$/MW); O&M fix., fixed operation and maintenance (US$/MW); O&M var., variable operation and maintenance (US$/MWh)

Table 4 Risk of cost in investment and operation and maintenance

	Combined cycle standard	IGCC	Solar photovoltaic	Hydro	Geothermal
invest.	5.73	14.60	0.00	0.00	0.00
O&M fix.	0.14	0.82	4.09	0.97	0.97
O&M var.	0.73	2.40	0.00	0.97	1.19

Invest., investment cost (US$/MW); O&M fix., fixed operation and maintenance (US$/MW); O&M var., variable operation and maintenance (US$/MWh)

Suppose I as the set of available technologies (index i), and K the set of cost categories (index k)

$$\mathrm{UTCO}_i = \sum_k C_{i,k} = \mathrm{INV}e_i + \mathrm{FU}_i + \mathrm{FOM}e_i + \mathrm{VOM}_i; \quad \forall i \in I \tag{16}$$

with

UTCO_i unit cost for technology i (US$/MWh).
$C_{i,k}$ component cost k for technology i.
$\mathrm{INV}e_i$ investment cost for technology i, expressed in (US$/MWh), per year (notation e expressed in terms of energy).
FU_i fuel cost for technology i.
$\mathrm{FOM}e_i$ fixed operation and maintenance costs for technology i expressed in (US$/MWh), per year (notation e expressed in terms of energy).
VOM_i variable operation and maintenance costs for technology i.

The total risk for the technology i consists of the risk of different costs of the category k, expressed by (18)

$$\sigma_i = \sqrt{\sum_k \sigma_{i,k}^2}, \quad \forall i \in I \tag{17}$$

With σ_i being the risk of cost to the technology i and $\sigma_{i,k}$ the risk of costs of category k and of technology i.

Correlations exist between the costs of different categories k and of different technologies i. The formulation of the correlation ρ_{ih} between the total cost of technologies i and h, and the cost of the components k and $l \in K$ is given by (19)

$$\rho_{ih} = \frac{\sum_k \sum_l \rho_{kl,ih} \cdot \sigma_{i,k}\sigma_{h,l}}{\sigma_i \cdot \sigma_h}, \quad \forall i \in I \tag{18}$$

With $\rho_{kl,ih}$ being the correlation between the cost of the categories k and l, to the technologies i and h.

The average cost of portfolio p, (denoted by $\mathrm{avcost_p}$) (US$/MWh), is defined by (20)

$$\mathrm{avcost_p} = \sum_i w_i \cdot \mathrm{UTCO}_i \tag{19}$$

With w_i being the weight of technology i in portfolio p. The risk of the portfolio is defined by (21), with $i, h \in I$

$$\sigma_\mathrm{p} = \sqrt{\sum_i \sum_h w_i \cdot w_h \cdot \rho_{ih} \cdot \sigma_i \cdot \sigma_h} \tag{20}$$

The portfolio of lower cost and lower risk for w_i can be determined by minimising (20) and (21) with the restrictions $\sum_i w_i = 1$ and $w_i \geq 0, \forall i \in I$.

3.2 Power generation scenario in California

The investment's fixed cost (construction or expansion of the plant), fixed operation and maintenance (human resources, administration and overhead) are expressed in US$/MW. The components of variable costs are fuel and variable operating and maintenance (repairs and auxiliary installations), expressed as US$/MWh.

According to [1, 5], the fuel cost is given by the variation of the market price. Based on the forecasts made in Energy Information Administration, [29], the time series of fuel costs can be obtained for plants that use natural gas and coal plants, since solar, hydro and geothermal use no fuel.

Fixed costs (investment, fixed operation and maintenance) and the variable operation and maintenance costs were based on [30] and are presented in Table 3.

The investment risk is determined by various parameters such as construction time, licensing, and permission of the environment regulator [1]. The risk of fuel is in line with price fluctuations over the years, so the variance is calculated for each replica [1, 5]. The risks of each component for investment and O&M fix. and O&M var. are given in (US$/MWh); they are shown in Table 4 and are based on [1, 5].

Correlations between components of similar costs (excluding fuel costs) are equal to 0.7, while the correlation between the different cost components is set to 0.1. These values are based on [1]. The correlation between fuel costs are calculated for each time series of each replica built from the moving windows.

3.3 Portfolio optimisation by MDE

After defining the scenario of power generation in the State of California, the optimum portfolio using the proposed methodology will be found using MDE with replicas and moving windows. This was achieved using the desirability function implemented in Minitab software and the entropy index was used to improve the diversity of the portfolio.

To address the seasonality and volatility of the time series that represent the cost of fuel, the concepts of moving windows and

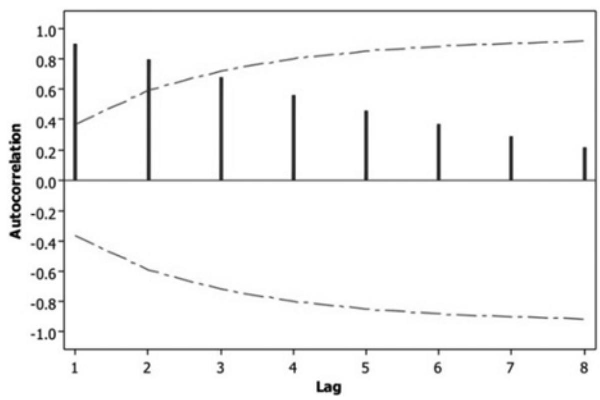

Fig. 4 *ACF at 5% significance of the cost of fuel to natural gas and coal*

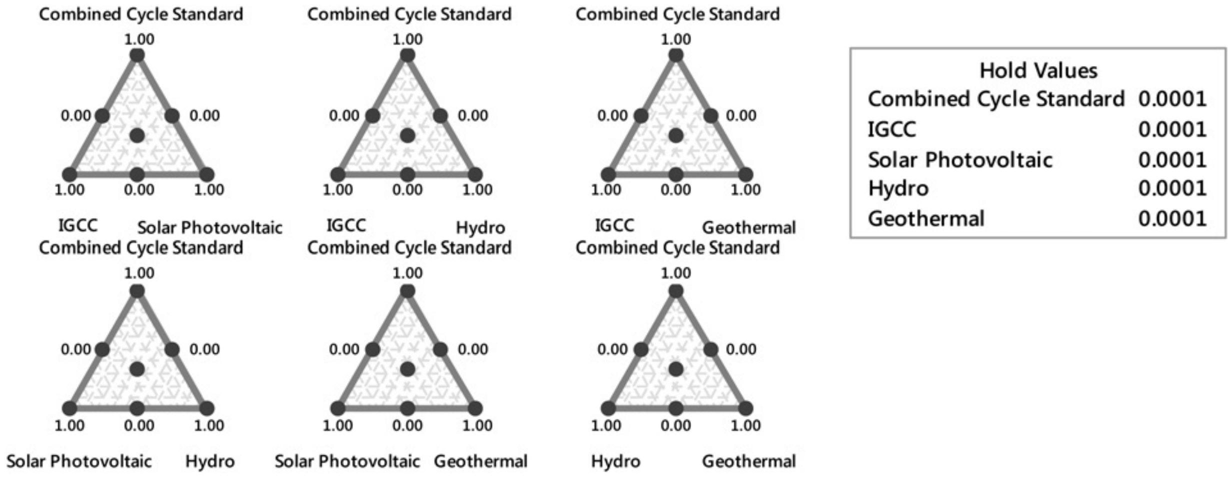

Fig. 5 *Arrangement mixtures of five technologies*

replicas in MDE were applied. For this, it was necessary to define the size and lag of the moving window (Step 1).

Of the 31 data points of the costs for natural gas and coal in the State of California, the window size was estimated using the definition in (11)

$$L = \frac{31}{4} = 7.75 \cong 8 \tag{21}$$

The lag is determined by the ACF, as described in Section 2.3. In Fig. 4, the ACF is presented for the cost of fuel for natural gas and coal. Fig. 4 illustrates that the lag of the ACF with 5% significance is 2. Therefore, the windows will move every two lags.

After defining the size and lag of the moving windows, they were applied to the 31 time series, resulting in the 13 replicas used in MDE.

Next, the cost ($UTCO_i$), risk (σ_i) and correlations (ρ_{ih}) for each component in each one of the 13 replicas were determined using (17), (18) and (19), respectively. The correlation between each replica was also calculated (Step 2).

Entropy was used to diversify the portfolio as stated on (8). The weights, w_i of technologies must not be zero. Therefore, the proportions were delimited between 0.01 and 99.96% for the lower and upper ends.

One can also add points within the design space (axial points). These points provide information about the interior of the response surface, improving the result [22]. Therefore, an axial point was chosen. Fig. 5 shows the constructed MDE (Step 3).

With the constructed MDE, the weights, w_i are used to calculate the responses of cost, risk and entropy, as can be seen in Table 5 (Step 4).

Subsequently, regression analysis is performed and a quadratic polynomial was chosen to model the cost and risk, according to (3) (Step 5).

Before finding the optimal portfolio using the desirability function, as defined in (14) and (15), it is necessary to define the parameters for the value of the Target (T_i), Upper Limit (H_i) and Lower Limit (L_i). These parameters are defined based on statistical values such as average, maximum and minimum cost value, risk and entropy (Step 6).

To minimise cost and risk, the Target (T_i) used was the minimum value and average was used as Upper Limit (H_i). To maximise entropy, the maximum value was chosen as the Target (T_i), while the average was used for the Lower Limit (L_i). These values are shown in Table 6.

Table 5 MDE with five components and 13 replicas in Minitab

	Combined cycle standard	IGCC	Solar photovoltaic	Hydro	Geothermal	Cost	Risk	Entropy
replica 1	0.9996	0.0001	0.0001	0.0001	0.0001	35.2556	5.7781	0.0018
	0.0001	0.9996	0.0001	0.0001	0.0001	95.0876	14.8086	0.0018
	0.0001	0.0001	0.9996	0.0001	0.0001	304.4723	4.0904	0.0018
	0.0001	0.0001	0.0001	0.9996	0.0001	109.6098	1.3691	0.0018
	0.0001	0.0001	0.0001	0.0001	0.9996	101.8637	1.5340	0.0018
	0.4999	0.4999	0.0001	0.0001	0.0001	65.1716	9.7295	0.3023
	0.4999	0.0001	0.4999	0.0001	0.0001	169.8640	3.7493	0.3023
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
replica 2	0.9996	0.0001	0.0001	0.0001	0.0001	35.2852	5.7794	0.0018
	0.0001	0.9996	0.0001	0.0001	0.0001	95.1227	14.8086	0.0018
	0.0001	0.0001	0.9996	0.0001	0.0001	304.4724	4.0904	0.0018
	0.0001	0.0001	0.0001	0.9996	0.0001	109.6098	1.3691	0.0018
	0.0001	0.0001	0.0001	0.0001	0.9996	101.8637	1.5340	0.0018
	0.4999	0.4999	0.0001	0.0001	0.0001	65.2039	9.7554	0.3023
	0.4999	0.0001	0.4999	0.0001	0.0001	169.8788	3.7498	0.3023
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
replica 13	0.9996	0.0001	0.0001	0.0001	0.0001	35.2852	5.7794	0.0018
	0.0001	0.9996	0.0001	0.0001	0.0001	95.1227	14.8086	0.0018
	0.0001	0.0001	0.9996	0.0001	0.0001	304.4724	4.0904	0.0018
	0.0001	0.0001	0.0001	0.9996	0.0001	109.6098	1.3691	0.0018
	0.0001	0.0001	0.0001	0.0001	0.9996	101.8637	1.5340	0.0018
	0.4999	0.4999	0.0001	0.0001	0.0001	65.2039	9.7554	0.3023
	0.4999	0.0001	0.4999	0.0001	0.0001	169.8788	3.7498	0.3023
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 6 Parameters used to maximise the desirability function

	Cost	Risk	Entropy
T_i	35.26	1.37	0.70
L_i			0.40
H_i	129.51	4.81	

Table 7 Optimal portfolio components

Combined cycle standard	IGCC	Solar photovoltaic	Hydro	Geothermal
32.32%	1.02%	0.01%	33.03%	33.62%

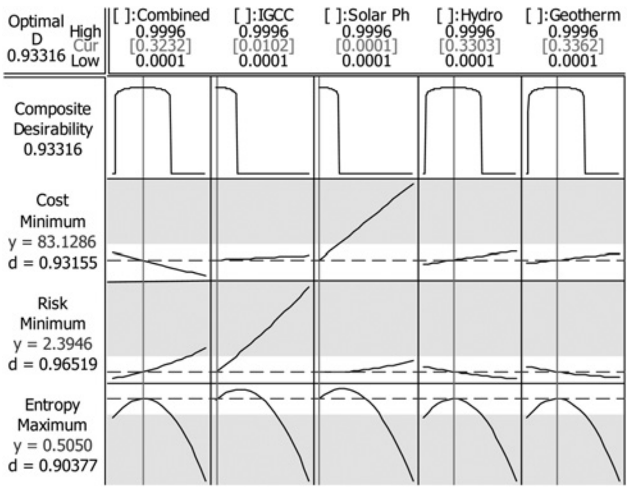

Fig. 6 *Optimal portfolio obtained through the desirability function*

The portfolio combination was achieved from these parameters and it is represented on Table 7. The desirability function is also presented on Fig. 6. These results correspond to Step 7 of the proposed methodology.

The value of the cost of the optimal portfolio found (Table 6) is US$ 83.13 by MWh and the risk is US$ 2.39 by MWh, as noted in Fig. 6.

To validate the proposed methodology, the obtained results were compared with the Markowitz mean-variance theory (blue points). An efficient frontier from MVP was traced. The optimal portfolio was achieved through the desirability function, represented by Point A in Fig. 7.

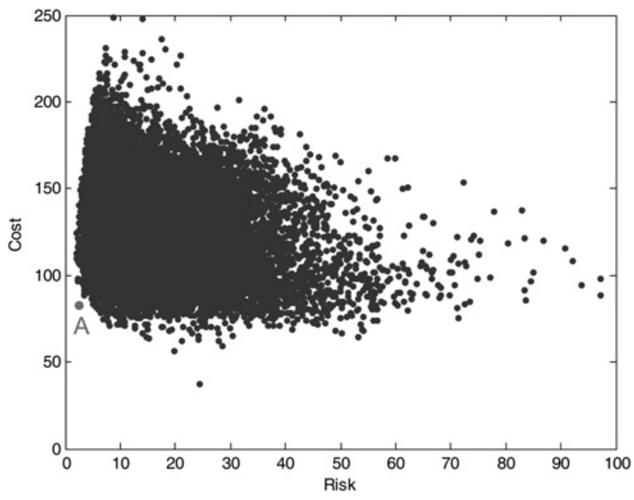

Fig. 7 *Efficient frontier determined by MVP. Point A is derived from Fig. 6*

As observed, Point A is very close to the feasible region and to the efficient frontier. Furthermore, the portfolio is located in the combination of lower cost and lower risk. Therefore, the result is satisfactory. In addition, the proposed methodology determines an optimal combination and not a set of optimal combinations as in MVP, which facilitates decision-making. So, for these five technologies (Combined Cycle Standard, IGCC, Solar Photovoltaic, Hydro and Geothermal), the combination found results in an optimised portfolio of lower cost and lower risk.

Another comparison was assessed for modelling time series by ARMA-GARCH with the proposed methodology. The results were statistically identical between the two methodologies for the t-test ($P-\text{value} = 0$). Due to statistical similarities, the advantage of using the methodology proposed is the simplicity (Occam's razor principle!), and processing time.

4 Conclusion

This study presented a new methodology to find the best combination for power generation to obtain a portfolio with the lowest cost and lowest risk. This combination, called an efficient portfolio, was determined using MDEs. This work also showed that Shannon entropy index along with replicas in MDE and moving windows can be used to come up with a diversified portfolio, dealing with seasonality and volatility of time series. The work also compared its results with the traditional theory of Markowitz mean-variance.

The advantage of using the proposed methodology with respect to MVP is that MVP produces a line (efficient frontier) with various possible solutions. In the proposed method, the result is an optimal combination of lower cost and lower risk, which facilitates decision making. Furthermore, with the use of moving windows and computational replicas in MDE, the time series modelling (which depends heavily on the expertise statistical judgment) is no more needed.

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